University of California, Berkeley
Physics H7C Fall 1999 (*Strovink*)

PROBLEM SET 6

Problems 1-4 are suitable review problems for Midterm 1. Problems 5-8 involve new material beyond the range covered by Midterm 1.

1.
Model an animal's eye as a sphere composed of vitreous humor, backed by a retina. If the eye is focused at infinity, what is the refractive index of the humor?

2. A point source of isotropic light is located at the center of a small hemispherical hole in the plane end face of a cylindrical light guide with refractive index $n = 2$, permeability $\mu = \mu_0$.

What fraction of the light emitted can be transmitted a long distance (relative to its radius) by the light guide? (A *number* is required.)

3.
Using a combination of optical devices (polarizers, wave plates...), design an optical system that will pass right-hand circularly polarized light without changing its polarization, but will completely block left-hand circularly polarized light. This system is called a "right-hand circular analyzer". Use Jones matrices to prove that your design will work.

4.
A Michelson interferometer produces fringes on its screen (which is not quite perfectly aligned). It is fed by laser light polarized out of the interferometer plane along $\hat{\mathbf{z}}$. With equal path lengths the fringe visibility $\mathcal{V} \equiv (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$ is unity.
(a.)
An ideal linear polarizer, with transmission axis oriented at 90° to $\hat{\mathbf{z}}$, is placed in one leg of the interferometer. What is $\mathcal{V}$? Explain.
(b.)
Same as (a.) except that a second linear polarizer, with transmission axis oriented at 45° to $\hat{\mathbf{z}}$, is added in the same leg *upstream* of the first (*i.e.* closer to the half-silvered mirror). Show your calculation for $\mathcal{V}$.

5.
Two identical horizontal thin slits in a black plate are centered at $y = \pm\frac{h}{2}$, where y is the vertical coordinate. A screen with vertical coordinate Y is located a distance D downstream. If an analyzer is present, it is located just upstream of the screen. Fraunhofer conditions apply, *i.e.* $kh^2 \ll D$, and small-angle approximations can be made, *i.e.* $|y| \ll D$, $|Y| \ll D$. Plane wave A is normally incident on the top slit and plane wave B is normally incident on the bottom slit, with

$$\mathbf{E}_A \propto \Re[(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{i(kz-\omega t)}]$$

and

$$\mathbf{E}_B \propto \Re[(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{i(kz-\omega t)}] \ ,$$

with $\Re$ denoting the real part. When either slit is blocked and no analyzer is in place, the intensity $I(Y = 0) \equiv I_0$. When neither slit is blocked, find $I(Y)/I_0$, where I_0 is defined above, for the following cases:
(a.)
No analyzer is in place.
(b.)
The analyzer accepts only $\hat{\mathbf{y}}$ polarized light.
(c.)
The analyzer accepts only right-hand circularly polarized light $\mathbf{E} \propto \Re[(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{i(kz-\omega t)}]$.

6.
Prove that

$$\sum_{n=1}^{N} \exp{(i\phi_n)} = \frac{\sin N\Delta\phi/2}{\sin \Delta\phi/2} \exp{(i\bar{\phi})},$$

where

$$\Delta\phi \equiv \phi_{n+1} - \phi_n,$$

and $\bar{\phi}$ is the average of the ϕ_n.

7.
A plane wave of wavelength λ is incident on (A)

no screen, *i.e.* all the light passes through; (B) a black disk of radius R; (C) a black screen with a circular hole of the same radius R. The relative intensities seen by an observer on the axis at distance D downstream are in the ratio $I_A : I_B : I_C = 1 : 1 : 0$. Fraunhofer conditions do not apply to this geometry, although the obliquity and $1/r$ factors do not vary appreciably across the screen.

(a.)

Find the smallest possible value of R that is consistent with the above conditions, expressed in terms of D and λ.

(b.)

In this problem the screen aperture functions g_B for case (B) and g_C for case (C) sum together to give the aperture function g_A for case (A). For the particular R that you obtained for part (a), the intensities I_B and I_C also sum together to give I_A. For what other choices of R would that be true? Explain.

8.

A plane electromagnetic wave propagates in the $\hat{\mathbf{z}}$ direction within a good conductor ($\lambda_{\text{EM wave}} \gg \delta(=\text{skin depth}) \gg \lambda_{\text{plasma}}$). Evaluate the *total* power lost per *square* meter due to Joule (ohmic) heating in the region $0 < z < \infty$. Show that this is equal to the average value of $|\mathbf{S}|$ at $z = 0$. You may take $\mu = \mu_0$ for this conductor.